is removed in energy by $\lesssim 0{\cdot}04$ eV from the heavy hole band, has an appropriate influence on hot hole velocities in p-type silicon above intermediate field values. Conduction processes in this band are in effect "turned on" when the fields reach the mid 10^4 V/cm range and this relatively low mass band becomes populated. If the velocity does saturate, our data suggests that it does so above 2×10^5 V/cm with a value $\sim 1{\cdot}0\times10^7$ cm/sec.

6. CONCLUSIONS

Hole velocities in uncompensated p-type silicon at 300°K have been found for electric fields to $1{\cdot}1\times10^5$ V/cm and for hole densities between 10^{14} and 10^{16} cm^{-3}. The measurements of the $J(V)$ relation and the analysis in terms of the rather small but important space charge injected current give the result that the $v(E)$ relation is essentially independent of the hole density. It is important that the contribution of SCIC is maintained small since it assures the meaningfulness of the study of the effect of doping. It also avoids the difficulties in obtaining accurate velocities from measurements of predominately space charge limited currents.(22)

The onset of impact ionization in our samples is only observed for fields $\bar{E} \gtrsim 10^5$ V/cm. The effective ionization rate computed from the observed ramp on the current pulse gives a resultant α which appears to agree with the ionization rates reported by LEE and co-workers.(15) The limitation of finding the velocity above 10^5 V/cm using our $J(V)$ technique is, therefore, understood.

Because of the care taken during the measurements to use an appropriate pulse length, the results are not influenced by displacement currents, by heating or by transient space charge effects. A method of contacting was used which removes the metal semiconductor contact from the immediate proximity of the p-region under study. This arrangement conveniently avoids the necessity of considering the detailed behavior of the metal-semiconductor contact. A nearly saturated drift velocity is observed, and by extrapolation, v_s appears to occur at or above 2×10^5 V/cm with a value between $0{\cdot}96\times10^7$ and $1{\cdot}06\times10^7$ cm/sec. The lack of a strong dependence of v upon hole density is consistent with STRATTON's conclusions.(4)

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THE ELASTIC CONSTANTS OF CADMIUM SULFIDE BETWEEN 4·2–300°K*

D. GERLICH†

RCA Laboratories, Princeton, N.J.

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Abstract—The five independent elastic constants of single crystal CdS have been determined over the temperature range 4·2–300°K. They were calculated from measurements of the sound velocity in different crystalline directions by the pulse superposition method. All elastic constants increase with decreasing temperature, the variation being somewhat larger for the cross coupling constants than for the diagonal ones. From the low temperature values of the elastic constant, the Debye temperature at 0°K for CdS is determined as 219·3°K.

1. INTRODUCTION

THE ELECTRO-ACOUSTIC properties of CdS have been of interest for a number of years,(1–4) especially since the discovery of the effect of ultrasonic amplification(2) in this material. Since all acousto-electric effects are intimately connected with the sound velocity in the material, the knowledge of this quantity is of great interest in these investigations. Room temperature values for the sound velocities and the elastic constants of CdS have been reported,(5–8) but no results are available for other temperatures. The present report describes measurements of sound velocity and the elastic constants of CdS single crystal between 4·2 and 300°K. From the low temperature elastic constants, the Debye temperature can be computed, and their temperature dependence will indicate the extent to which anharmonic effects are present in the CdS lattice.

Table 1. *Impurity contents of the* CdS *crystals*

(Amounts of impurities are in ppm by weight)

Impurity	CdS 3–125/1	CdS 3–125/2
Na	1000–10,000	1000–10,000
Ca	20–200	3–30
Al	100–1000	3–30
Pb	3–30	3–30
Mg	3–60	3–30
Si	30–300	1–10
Fe	30–300	10–100
In	100–1000	30–300
Cu	100–1000	30–300
Ag	30–300	30–300
Zn	100–1000	30–300

2. EXPERIMENTAL TECHNIQUES

Two high purity single crystals of CdS were used in the present investigation. Results of emission spectrographic analysis done on these crystals are presented in Table 1. The crystals were oriented by means of X-ray Laue back reflection, and then several planes were ground on each of them. On one of the crystals two pairs of parallel faces were ground, one set normal to the c axis, the other normal to the a axis. On the second crystal, one pair of parallel faces was ground, their normal being in the ac plane, and making an angle of 45° with each of the a and c axis. All faces were polished optically flat, and parallel, within a few seconds of arc.

The elastic constants were determined by measuring the velocity of ultrasonic waves in CdS.

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† Present address: Physics Department, Tel Aviv University, Ramat Aviv, Israel.

Measurements were made in the dark to avoid effects of photoconductivity.(9,10) As the CdS crystals were of high resistivity, electrical conductivity effects could be neglected. Since CdS is a piezoelectric material the electrical boundary conditions for the elastic constants have to be specified. In addition the piezoelectric properties cause a "stiffening" of the lattice(11,12) which has to be taken into account when relating the sound velocity to the elastic constants.

Cadmium sulfide crystallizes in the hexagonal system, and thus it has five independent elastic constants c_{11}, c_{12}, c_{13}, c_{33}, and c_{44}. For the determination of the latter eight different propagation modes of the sound are available for velocity determination. These modes together with the relations between the sound velocities and the elastic constants are listed in Table 2. Here the c_{ij}'s are the adiabatic elastic constants at *constant electric field*, the e_{ij}'s and ϵ_i's the piezoelectric and adiabatic dielectric constants respectively, ρ the density, and the conductivity has been assumed to be zero. The convention of axis is that the z axis is parallel to the crystalline c axis, the x axis to the a axis, while the y axis is normal to the x and z axis, the three forming a right handed system.

The sound velocity was measured by the McSkimin pulse superposition method.(13,14) The longitudinal and shear waves were generated by X and Y cut crystalline quartz transducers respectively, operating at their fundamental frequency of 15 Mc/s. Over the temperature range of 78–300°K, Canada balsam and Dow Corning DC 200 silicone fluid (viscosity 12,500 cS) were used for bonding the transducer to the crystal, while for the range 4·2–78°K 4-Methylpentene-1 was the bonding agent.

In order to compute the elastic constants at temperatures different from room temperature, a correction for the change in path length and density has to be applied, this correction requiring the knowledge of the thermal expansion coefficient as a function of temperature. Since such data for the range 4·2–3000°K are not available, this coefficient was estimated from the room temperature value,(15) assuming its temperature dependence is a Debye function. For the 45° direction the expansion coefficient was taken as $\frac{1}{2}(\alpha_1+\alpha_3)$ where α_1, and α_3 are the expansion coefficients in the a and c direction, thus neglecting the change in angle with thermal expansion. Since the correction due to thermal expansion is very small, such an estimate is considered to be adequate. As only the room temperatures values of the ϵ_i's and e_{ij}'s for CdS are known,(8) the latter were used over the whole temperature range. This will not introduce an appreciable error as the correction due to the term containing the ϵ_i's and e_{ij}'s adds a correction of about 2 per cent. By analogy with zinc sulfide, the variation of the ϵ_i's and e_{ij}'s is of the order of 2 per cent over the temperature range room—78°K. Thus the neglect of the temperature dependence of the e_{ij}'s and the ϵ_i's will introduce an error of the order of 0·5 per cent.

Table 2. The relation between the sound velocity and the elastic constants for CdS *single crystal*

Velocity	Direction of propagation	Mode of propagation	Relation between sound velocity and elastic constants
v_1	$z = \parallel c$ axis	Longitudinal	$\rho v_1^2 = c_{33}+e_{33}^2/\epsilon_3$
v_2	$z = \parallel c$ axis	Shear, polarized in z plane	$\rho v_2^2 = c_{44}$
v_3	$x = \parallel a$ axis	Shear, polarized $\parallel z$	$\rho v_3^2 = c_{44}+e_{31}e_{15}/\epsilon_1$
v_4	$x = \parallel a$ axis	Shear, polarized $\perp z$	$\rho v_4^2 = (c_{11}-c_{12})/2$
v_5	$x = \parallel a$ axis	Longitudinal	$\rho v_5^2 = c_{11}$
v_6	45° to a and c axis	Shear, polarized $\parallel y$	$\rho v_6^2 = (c_{11}-c_{12}+2c_{44})/4$
v_7	45° to a and c axis	Quasi longitudinal	$\rho v_7^2 = (c_{11}^1+c_{55}^1)/2+[(c_{11}^1-c_{55}^1)^2+4c_{15}^1c_{51}^1]^{1/2}/2$
v_8	45° to a and c axis	Quasi shear	$\rho v_8^2 = (c_{11}^1+c_{55}^1)/2-[(c_{11}^1-c_{55}^1)^2+4c_{15}^1c_{51}^1]^{1/2}/2$

$c_{11}^1 = (c_{11}+c_{33}+2c_{13}+4c_{44})/4+(2e_{15}+e_{31}+e_{33})^2/[2(\epsilon_1+\epsilon_3)]$

$c_{15}^1 = (c_{11}-c_{33})/4+(2e_{15}+e_{31}+e_{33})(e_{31}-e_{33})/[2(\epsilon_1+\epsilon_3)]$

$c_{55}^1 = (c_{11}+c_{33}-2c_{13})/4+(e_{31}-e_{33})(2e_{15}-e_{31}-e_{33})/[2(\epsilon_1+\epsilon_3)]$

$c_{51}^1 = (c_{11}-c_{33})/4+(2e_{15}+e_{31}+e_{33})(2e_{15}-e_{31}-e_{33})/[2(\epsilon_1+\epsilon_3)]$

The ultimate accuracy which can be achieved with the pulse super-position method is of the order of 1 in 10^5. Such an accuracy is however conditioned by a perfect echo pattern. Due to structural imperfection in the crystals, slight misalignment from the true crystalline direction and other disturbances, a perfect echo pattern could not be achieved, and thus the ultimate accuracy of the measuring method could not be realized. It is estimated that the accuracy of the sound velocity determination in the present measurements is about 1 in 1000.

3. RESULTS AND DISCUSSION

The measured eight different sound velocities as a function of the temperature over the range 4·2–300°K are presented in Figs. 1 and 2. The five elastic constants were determined from the measured values of v_1, v_2, v_3, v_4, v_5 and v_8; while v_6 and v_7 served as a check on the consistency of the results. The elastic constants as a function of temperature are shown in Figs. 3 and 4. The former figure presents the diagonal c_{11}, c_{33}, and c_{44}, while the latter showing the cross coupling constants c_{12} and c_{13}, a room temperature density of 4·824 g cm^{-3} being used in the computation. As can be seen the cross coupling constants vary somewhat more with temperature than the diagonal constants. Overall, the variation of the elastic constants with temperature is quite small, which indicates that anharmonic effects in the CdS lattice are small. This is also corroborated by the low values of the thermal expansion coefficient.(15)

Table 3 shows a comparison of the present room temperature elastic constants with results of other investigators. As can be seen the agreement with Bolef *et al.* for the values of c_{11}, c_{12} and c_{13} is very good while there is a discrepancy for c_{33} and c_{44}. This discrepancy seems to be due to neglect of the piezoelectric correction by Bolef *et al.* c_{11} and c_{12} are not affected at all by this correction while c_{14} only very slightly.

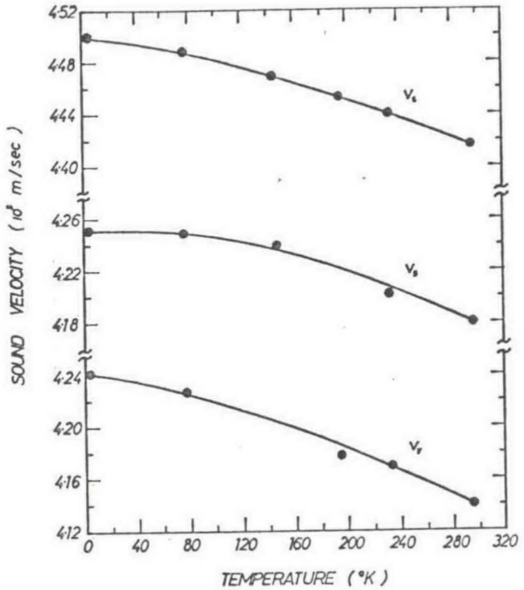

Fig. 1. Sound velocity for longitudinal waves in different crystalline directions as a function of temperature.

From the low temperature elastic constants, the Debye temperature at 0°K can be determined.

Table 3. Comparison of the present room temperature elastic constants of CdS *with former work*

	c_{11} 10^{10} N/m²	c_{33} 10^{10} N/m²	c_{44} 10^{10} N/m²	c_{12} 10^{10} N/m²	c_{13} 10^{10} N/m²
Bolef *et al.*(6)	8·432	9·397	1·489	5·212	4·638
Berlincourt *et al.*(8)	9·07	9·38	1·504	5·81	5·10
Present work	8·431	9·183	1·458	5·208	4·567

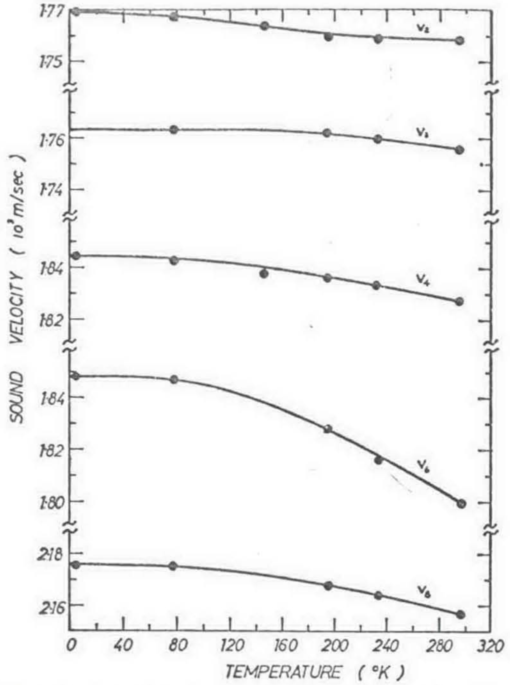

FIG. 2. Sound velocity for shear waves in different crystalline directions as a function of temperature.

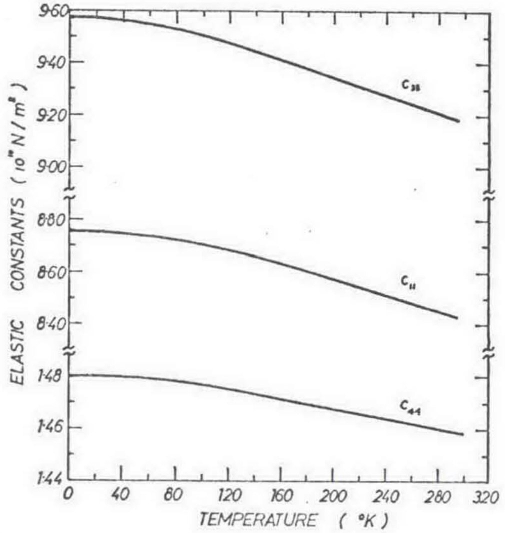

FIG. 3. The diagonal elastic constants as a function of temperature.

Cadmium sulfide is very close to being an isotropic material, as the values of the ratios c_{11}/c_{33}, c_{12}/c_{13} and $(2c_{44})/(c_{11}-c_{12})$ are all close to 1. Hence, the procedure devised by ANDERSON[16] for evaluating the Debye temperature from the isotropic Voigt–Reuss–Hill–Gilvarry average of the elastic constants is certainly applicable in the present case. Proceeding in this manner, the Debye temperature at 0°K is calculated to be 219·3°K. Unfortunately, low temperature specific heat data for CdS are not available, and thus no comparison can be made with the Debye temperature determined from specific heat data.

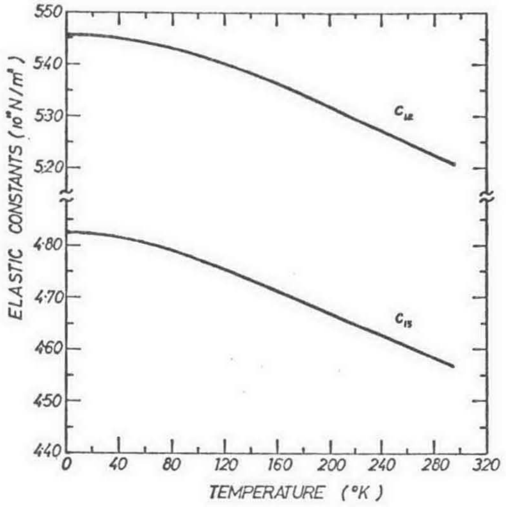

FIG. 4. The cross coupling elastic constants as a function of temperature.

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